EECS 861
Homework 11

1. In a target detection problem, the target is present for 100 μs. When the target is not present only noise N(t) is received. N(t) is additive zero mean Gaussian WSS random process, N(t) has the following PSD

$$S_N(f)=\begin{cases}\dfrac{\eta}{2}=10^{-5} & |f|<50\text{kHz}\\ 0 & \textit{elsewhere}\end{cases}$$

When the target is present the received signal is Y(t)=A+ N(t) where A=1.5. Y(t) is sampled every 10 μs. One sample of Y(t) are used to detect the presence of the target.

   a. Assuming P(target is present)= P(target is not present)=.5 derive the MAP decision rule.
   b. Find P_D, P_M, and P_{fa} given the MAP decision rule.
   c. Design an N-P detector is to obtain a P_{fa} =0.01
   d. Find P_D, P_M, and P_{fa} given the N-P detector.

2. Chapter 6: Problem 6.4

3. During a bit time T_B of 1 sec the transmitted signal X(t) is –A V (bit=0) or AV (bit=1) where A=0.5 and bits are transmitted with equal probability. The transmitted signal is corrupted by additive zero mean WSS random process, where N(t) has the following PSD. The received signal is Z(t)=X(t)+N(t)

$$S_N(f)=\begin{cases}\dfrac{\eta}{2}=\dfrac{20}{100} & |f|<50\\ 0 & \textit{elsewhere}\end{cases}$$

The decision variable, Y is given by

$$Y=\frac{1}{T_B}\int_0^{T_B} Z(t)dt$$

a) Find the distribution of Y|0 bit is transmitted.
b) Find the distribution of Y|1 bit is transmitted.
c) Derive the MAP decision rule.
d) Find the probability of bit error, P_e.

4. Trade-offs
    a. Will the P_D increase or decrease as A increases in Problem 1 with the N-P detector?
    b. Will the P_{fa} increase or decrease as A increases in Problem 1 with the N-P detector?
    c. Will the P_e increase or decrease as A increases in Problem 3?
    d. What is the system cost for increasing A in Problem 3?
    e. Will the P_e increase or decrease as η increases in Problem 3?

5. Chapter 6: Problem 6.12 Verify your answer using http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/ROC.cdf or http://demonstrations.wolfram.com/SignalDetectionTheory/